A Level Pure Mathematics
Practice Test 4: Trigonometry

Instructions:
Answer all questions. Show your working clearly.
Calculators may be used unless stated otherwise.
Time allowed: 2 hours

Section A: Radian Measure and Basic Functions

1. Convert these angles from degrees to radians (leave answers in terms of π):
   (a) $24°$
   (b) $40°$
   (c) $144°$
   (d) $225°$
   (e) $315°$
   (f) $330°$
2. Convert these angles from radians to degrees:
   (a) $\frac{\pi}{15}$
   (b) $\frac{2\pi}{9}$
   (c) $\frac{7\pi}{8}$
   (d) $\frac{9\pi}{4}$
   (e) $\frac{10\pi}{3}$
   (f) $\frac{17\pi}{6}$
3. Find the exact values of these trigonometric ratios (without calculator):
   (a) $\sin(\frac{3\pi}{2})$, $\cos(\frac{3\pi}{2})$, $\tan(\frac{3\pi}{2})$
   (b) $\sin(\frac{5\pi}{2})$, $\cos(\frac{5\pi}{2})$, $\tan(\frac{5\pi}{2})$
   (c) $\sin(-\pi)$, $\cos(-\pi)$, $\tan(-\pi)$
   (d) $\sin(-\frac{3\pi}{2})$, $\cos(-\frac{3\pi}{2})$, $\tan(-\frac{3\pi}{2})$
4. A circle has radius 20 cm. Find:
   (a) The arc length subtended by an angle of $\frac{7\pi}{8}$ radians
   (b) The area of the sector with angle $\frac{5\pi}{9}$ radians
   (c) The angle (in radians) that subtends an arc of length 35 cm
   (d) The radius of a circle where an angle of $\frac{4\pi}{5}$ radians subtends an arc of length 48 cm
5. Find the exact values:

(a) $\sin \frac{7\pi}{6}$
(b) $\cos \frac{7\pi}{4}$
(c) $\tan \frac{4\pi}{3}$
(d) $\sin \frac{8\pi}{3}$
(e) $\cos \frac{9\pi}{4}$
(f) $\tan \frac{13\pi}{6}$

Section B: Graphs of Trigonometric Functions

6. For the function $f(x) = 3\sin x$:
   (a) State the domain and range
   (b) Find the period
   (c) Identify the zeros in the interval $[0, 2\pi]$
   (d) Find the maximum and minimum values and where they occur
   (e) Sketch the graph for $x \in [-2\pi, 2\pi]$

7. For the function $g(x) = \cos \frac{x}{3}$:
   (a) State the domain and range
   (b) Find the period
   (c) Identify the zeros in the interval $[0, 6\pi]$
   (d) Find the maximum and minimum values and where they occur
   (e) Sketch the graph for $x \in [0, 6\pi]$

8. For the function $h(x) = \tan 3x$:
   (a) State the domain and range
   (b) Find the period
   (c) Identify the asymptotes in the interval $[0, \pi]$
   (d) Find the zeros in the interval $[0, \frac{\pi}{3}]$
   (e) Sketch the graph for $x \in [0, \frac{2\pi}{3}]$

9. Sketch the graphs of these transformed functions for $x \in [0, 4\pi]$:
   (a) $y = 4\sin x$
   (b) $y = \cos \frac{x}{4}$
   (c) $y = \sin(x + \frac{\pi}{4})$
   (d) $y = \cos x + 3$
   (e) $y = -2\cos x$
   (f) $y = \tan(x - \frac{\pi}{3})$

10. For the function $y = 5\cos(4x - \frac{\pi}{2}) + 3$:
   (a) Identify the amplitude
   (b) Find the period
   (c) Determine the phase shift
   (d) Find the vertical shift
   (e) State the range
   (f) Sketch the graph for $x \in [0, \pi]$

Section C: Fundamental Trigonometric Identities

11. Use the Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$ to find:
    (a) $\sin\theta$ if $\cos\theta = \frac{9}{41}$ and θ is acute
    (b) $\cos\theta$ if $\sin\theta = -\frac{8}{17}$ and θ is in the fourth quadrant
    (c) $\tan\theta$ if $\cos\theta = \frac{15}{17}$ and $\sin\theta > 0$
    (d) $\sin\theta$ if $\tan\theta = -\frac{20}{21}$ and $\cos\theta < 0$

12. Prove these quotient identities:
    (a) $\tan\theta = \frac{\sin\theta}{\cos\theta}$
    (b) $\cot\theta = \frac{\cos\theta}{\sin\theta}$
    (c) $\sec\theta = \frac{1}{\cos\theta}$
    (d) $\csc\theta = \frac{1}{\sin\theta}$

13. Simplify these expressions:
    (a) $\sin^2\theta(1 + \cot^2\theta)$
    (b) $\frac{\sec\theta}{\tan\theta} + \frac{\csc\theta}{\cot\theta}$
    (c) $(\tan\theta + \cot\theta)^2$
    (d) $\frac{\sec^2\theta - 1}{\tan\theta}$

14. Express in terms of $\sec\theta$ only:
    (a) $\cos^2\theta$
    (b) $\sin^2\theta$
    (c) $\tan^2\theta$
    (d) $\cos^2\theta + \sin^2\theta\csc^2\theta$

15. Prove that:
    (a) $\frac{\tan^2\theta}{\sec\theta - 1} = \sec\theta + 1$
    (b) $\sec^4\theta - \tan^4\theta = 1 + 2\tan^2\theta$
    (c) $\frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} = \frac{\tan\theta + 1}{\tan\theta - 1}$
    (d) $(1 + \sin\theta)(1 - \sin\theta) = \cos^2\theta$

Section D: Addition and Double Angle Formulas

16. Use the addition formulas to find the exact values:
    (a) $\sin 165°$ (using $\sin(120° + 45°)$)
    (b) $\cos 165°$ (using $\cos(120° + 45°)$)
    (c) $\tan 195°$ (using $\tan(240° - 45°)$)
    (d) $\cos\frac{11\pi}{12}$ (using $\cos(\frac{2\pi}{3} + \frac{\pi}{4})$)

17. Given $\cos A = \frac{8}{17}$ with A acute and $\sin B = \frac{15}{17}$ with B acute:
    (a) Find $\sin A$ and $\cos B$
    (b) Calculate $\sin(A + B)$
    (c) Calculate $\cos(A - B)$
    (d) Find $\tan(2A - B)$

18. Use double angle formulas to find:
   (a) $\cos 2\theta$ if $\sin\theta = \frac{12}{13}$ and θ is acute
   (b) $\sin 2\theta$ if $\cos\theta = \frac{20}{29}$ and θ is acute
   (c) $\tan 2\theta$ if $\tan\theta = \frac{7}{24}$
   (d) $\sin 2\theta$ if $\cos\theta = -\frac{9}{41}$ and θ is in the third quadrant

19. Prove these power reduction formulas:
   (a) $\sin^2\theta = \frac{1-\cos 2\theta}{2}$
   (b) $\cos^2\theta = \frac{1+\cos 2\theta}{2}$
   (c) $\tan^2\theta = \frac{1-\cos 2\theta}{1+\cos 2\theta}$
   (d) $\sin^4\theta = \frac{3-4\cos 2\theta+\cos 4\theta}{8}$

20. Express in reduced form:
   (a) $4\sin^2\theta$ in terms of $\cos 2\theta$
   (b) $4\cos^2\theta$ in terms of $\cos 2\theta$
   (c) $\sin^2\theta\cos^2\theta$ in terms of $\cos 4\theta$
   (d) $\cos^4\theta$ in terms of $\cos 2\theta$ and $\cos 4\theta$

Section E: Solving Trigonometric Equations

21. Solve these equations for $0 \leq x \leq 2\pi$:
   (a) $\cos x = \frac{1}{2}$
   (b) $\sin x = -\frac{\sqrt{2}}{2}$
   (c) $\tan x = \sqrt{3}$
   (d) $\cos x = -\frac{\sqrt{3}}{2}$

22. Solve these equations for $0° \leq x \leq 360°$:
   (a) $3\cos x - 1 = 0$
   (b) $2\sin x + \sqrt{2} = 0$
   (c) $\tan x - \frac{1}{\sqrt{3}} = 0$
   (d) $4\sin^2 x = 3$

23. Solve these equations for $0 \leq x \leq 2\pi$:
   (a) $\cos 3x = -\frac{\sqrt{3}}{2}$
   (b) $\sin 4x = \frac{1}{2}$
   (c) $\tan\frac{x}{2} = 1$
   (d) $\cos(x + \frac{\pi}{2}) = -\frac{1}{2}$

24. Solve these quadratic trigonometric equations for $0 \leq x \leq 2\pi$:
   (a) $3\cos^2 x + 2\cos x - 1 = 0$
   (b) $2\sin^2 x - 3\sin x + 1 = 0$
   (c) $\tan^2 x - 3\tan x + 2 = 0$
   (d) $5\cos^2 x - 6\cos x + 1 = 0$

25. Solve these equations involving multiple angles for $0 \leq x \leq 2\pi$:
    (a) $\sin x = -\cos x$
    (b) $\sin 2x = \cos x$
    (c) $\cos 2x = \sin x$
    (d) $\sin 4x = \sin 2x$

Section F: Advanced Trigonometric Identities

26. Prove these Werner formulas:
    (a) $\sin A \sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$
    (b) $\cos A \cos B = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$
    (c) $\sin A \cos B = \frac{1}{2}[\sin(A+B) + \sin(A-B)]$
    (d) $\cos A \sin B = \frac{1}{2}[\sin(A+B) - \sin(A-B)]$
27. Use Werner formulas to simplify:
    (a) $\sin 8x \sin 2x$
    (b) $\cos 9x \cos 3x$
    (c) $\sin 85^\circ \cos 25^\circ$
    (d) $\cos 110^\circ \sin 50^\circ$
28. Derive these Weierstrass substitutions where $u = \tan \frac{x}{2}$:
    (a) $\sin x = \frac{2u}{1+u^2}$
    (b) $\cos x = \frac{1-u^2}{1+u^2}$
    (c) $\tan x = \frac{2u}{1-u^2}$
    (d) $dx = \frac{2du}{1+u^2}$
29. Transform using Weierstrass substitution:
    (a) $5 \sin x - 3 \cos x$
    (b) $2 \sin x + \cos x$
    (c) $\frac{\cos x}{1+\sin x}$
    (d) $\frac{1}{3+2\cos x}$
30. Prove the quintuple angle formulas:
    (a) $\sin 5\theta = 16 \sin^5 \theta - 20 \sin^3 \theta + 5 \sin \theta$
    (b) $\cos 5\theta = 16 \cos^5 \theta - 20 \cos^3 \theta + 5 \cos \theta$
    (c) $\tan 5\theta = \frac{5\tan\theta - 10\tan^3\theta + \tan^5\theta}{1 - 10\tan^2\theta + 5\tan^4\theta}$

Section G: Complex Trigonometric Problems

31. Solve these equations for $0 \leq x < 2\pi$:
    (a) $\sin x + \cos x = \frac{1}{2}$
    (b) $\cos x + \cos 3x = 0$
    (c) $\sin x + \sin 3x + \sin 5x = 0$
    (d) $\sec x + \sec 2x = 0$

32. Prove these sophisticated identities:

(a) $\frac{\sin 5\theta}{\sin\theta} - \frac{\cos 5\theta}{\cos\theta} = 4\sin 4\theta$

(b) $\tan^2\theta + \tan^2(\theta + \frac{\pi}{3}) + \tan^2(\theta - \frac{\pi}{3}) = 9$

(c) $\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta$

(d) $\sin^2 A + \sin^2 B - \sin^2 C = 2\sin A\sin B\cos C$ when $A + B + C = \pi$

33. Find the general solution to these equations:

(a) $\sin x = \frac{4}{5}$

(b) $\cos 5x = -0.8$

(c) $\tan\frac{x}{3} = 4$

(d) $\sin(5x - \frac{\pi}{2}) = \frac{\sqrt{3}}{2}$

34. Express these in the form $R\sin(x + \alpha)$ or $R\cos(x + \alpha)$:

(a) $20\sin x + 21\cos x$

(b) $9\sin x - 40\cos x$

(c) $6\sin x + 6\cos x$

(d) $5\cos x - 5\sqrt{3}\sin x$

35. Find the range of these functions:

(a) $f(x) = 20\sin x + 21\cos x$

(b) $g(x) = 9\sin 4x - 40\cos 4x + 10$

(c) $h(x) = \cos^2 x + 5\sin x$

(d) $k(x) = 8\sin x\cos x - 3$

Section H: Applications of Trigonometry

36. A mass on a spring oscillates with displacement $s = 12\cos(5t - \frac{\pi}{4})$ millimeters, where t is time in seconds.

(a) Find the amplitude of the motion

(b) Determine the period of oscillation

(c) Find the phase shift

(d) Calculate the displacement when $t = 0$

(e) Find when the mass first passes through equilibrium

37. The height of the sun above the horizon is modeled by $h(t) = 45\sin(\frac{\pi t}{12} + \frac{\pi}{6}) + 30$ degrees, where t is hours after sunrise.

(a) Find the maximum and minimum heights

(b) Determine the period of the cycle

(c) Find the height 4 hours after sunrise

(d) Calculate when the sun reaches maximum height

(e) Find when the height is exactly 60 degrees

38. A sound wave is given by $p = 0.005\sin(440 \times 2\pi t + \frac{\pi}{3})$ pascals.

(a) Find the maximum pressure amplitude

(b) Determine the frequency of the sound

(c) Calculate the pressure when $t = 0.001$ seconds
(d) Find when the pressure first equals 0.003 pascals
(e) Determine what musical note this represents

39. A weather vane oscillates with angular displacement $\phi = 0.3\sin(2t + \frac{\pi}{3})$ radians, where t is time in seconds.

(a) Find the maximum angular displacement
(b) Determine the period of oscillation
(c) Calculate the angular displacement at $t = 0$
(d) Find when the vane first reaches maximum displacement
(e) Determine the angular velocity at $t = \frac{\pi}{4}$ seconds

40. Two light waves with equations $I_1 = 5\sin 4x$ and $I_2 = 12\cos 4x$ interfere.

(a) Find the equation of the resultant intensity
(b) Express the result in the form $R\cos(4x - \alpha)$
(c) Determine the amplitude of the resultant wave
(d) Find the phase relationship between the original waves
(e) Calculate the positions of maximum intensity

Section I: Advanced Problem Solving

41. In triangle DEF, $d = 13$, $e = 20$, and $\angle F = 75°$.

(a) Use the cosine rule to find side f
(b) Use the sine rule to find $\angle D$
(c) Calculate the area of the triangle
(d) Find the radius of the escribed circle opposite vertex F
(e) Determine the length of the angle bisector from F

42. Prove that in any triangle DEF:

(a) $\frac{d}{\sin D} = \frac{e}{\sin E} = \frac{f}{\sin F} = 2R$ (sine rule)
(b) $d^2 = e^2 + f^2 - 2ef\cos D$ (cosine rule)
(c) $\cot D + \cot E + \cot F = \cot D \cot E \cot F$
(d) Area $= \frac{def}{4R}$ where R is circumradius

43. A regular decagon is inscribed in a circle of radius r.

(a) Find the central angle for each sector
(b) Calculate the side length of the decagon
(c) Find the area of the decagon
(d) Determine the diagonal lengths
(e) Calculate the golden ratio relationships in the decagon

44. The function $j(x) = s\sin 3x + t\cos 3x$ has maximum value 25 and minimum value -25.

(a) Express $j(x)$ in the form $R\sin(3x + \delta)$
(b) Find the relationship between s and t
(c) If $j(\frac{\pi}{9}) = 20$, find the values of s and t

(d) Solve $j(x) = 15$ for $0 \leq x \leq \frac{2\pi}{3}$

(e) Find the values of x where $j''(x) = 0$

45. Consider the identity relating to regular heptagons: $\cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7} = -\frac{1}{2}$.

(a) Verify this identity using complex number methods

(b) Use this to find $\sin\frac{2\pi}{7} + \sin\frac{4\pi}{7} + \sin\frac{6\pi}{7}$

(c) Find the exact value of $\cos\frac{2\pi}{7}$

(d) Express the side length of a regular heptagon in terms of its circumradius

(e) Use these results to construct approximate vertices of a regular heptagon

Answer Space

Use this space for your working and answers.

END OF TEST

Total marks: 150

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