A Level Mechanics
Practice Test 4: Momentum and Impulse

Instructions:
Answer all questions. Show your working clearly.
Calculators may be used unless stated otherwise.
Draw diagrams where appropriate to illustrate your solutions.
Time allowed: 2 hours 30 minutes

Section A: Advanced Momentum Theory [35 marks]

Question 1 [**20 marks**] A system consists of five particles arranged as follows: A (mass 4 kg at position (2, 3) m with velocity (5, -2) m/s), B (mass 6 kg at position (-1, 4) m with velocity (-3, 4) m/s), C (mass 2 kg at position (3, -2) m with velocity (0, 6) m/s), D (mass 3 kg at position (-2, -1) m with velocity (4, -3) m/s), and E (mass 5 kg at position (1, 0) m with velocity (-2, 1) m/s).

(a) Calculate the total momentum of the system. [**4 marks**]

(b) Find the position and velocity of the center of mass. [**6 marks**]

(c) If an external force $(20\hat{i} - 15\hat{j})$ N acts for 4 seconds, find the new velocity of the center of mass. [**5 marks**]

(d) Calculate the impulse delivered to the system and verify using the change in total momentum. [**5 marks**]

Question 2 [**15 marks**] A spacecraft of mass 3000 kg traveling at 1200 m/s needs to change its velocity vector by 400 m/s perpendicular to its current direction without changing speed.

(a) Calculate the required change in momentum vector. [**4 marks**]

(b) If the spacecraft achieves this by ejecting fuel at 2500 m/s relative to the spacecraft, calculate the mass of fuel that must be ejected. [**6 marks**]

(c) Find the direction in which the fuel must be ejected relative to the spacecraft's initial velocity. [**5 marks**]

Section B: Complex Collision Sequences [40 marks]

Question 3 [**25 marks**] A series of collisions occurs on a frictionless track. Ball A (mass 3 kg, velocity 8 m/s) collides with ball B (mass 4 kg, velocity 2 m/s, same direction). After this collision, ball B then collides with stationary ball C (mass 5 kg). All collisions have coefficient of restitution 0.8.

(a) Calculate the velocities of A and B after the first collision. [**6 marks**]

(b) Find the velocities of B and C after the second collision. [**6 marks**]

(c) Calculate the total kinetic energy lost in both collisions. **[6 marks]**

(d) Determine the impulse experienced by ball B during both collisions. **[4 marks]**

(e) Find the time taken for both collisions if each contact lasts 0.01 seconds and ball B travels 0.5 m between collisions at its post-first-collision velocity. **[3 marks]**

Question 4 [15 marks] A pendulum bob of mass 2.5 kg hangs from a 2 m string. A bullet of mass 0.02 kg moving horizontally at 450 m/s embeds in the bob. The combined mass then swings and, at the bottom of its swing, collides elastically with a stationary block of mass 1.5 kg.

(a) Calculate the velocity of the bob immediately after the bullet embeds. **[3 marks]**

(b) Find the maximum angle the pendulum makes with the vertical. **[4 marks]**

(c) Calculate the velocity of the bob at the bottom of its swing just before hitting the block. **[3 marks]**

(d) Find the velocities of both the bob and block after their elastic collision. **[5 marks]**

Section C: Non-Uniform Force Analysis [30 marks]

Question 5 [18 marks] A force acting on a 1.2 kg particle varies according to $F(t) = 24t - 8t^2 + 2t^3$ N for $0 \leq t \leq 4$ seconds. The particle starts from rest.

(a) Calculate the impulse delivered during the entire 4-second interval. **[4 marks]**

(b) Find the final velocity of the particle. **[3 marks]**

(c) Determine when the force reaches its maximum value and calculate this maximum force. **[5 marks]**

(d) Calculate the impulse delivered from t = 0 to the time of maximum force. **[3 marks]**

(e) Find the velocity of the particle when the force is maximum. **[3 marks]**

Question 6 [12 marks] A golf ball of mass 0.045 kg is struck by a club. The force varies as shown: F = 0 for t 0, F = 1000t N for 0 t 0.002 s, F = 2 - 1000t N for 0.002 t 0.004 s, and F = 0 for t 0.004 s.

(a) Sketch the force-time graph. **[2 marks]**

(b) Calculate the total impulse delivered to the ball. **[4 marks]**

(c) If the ball was initially at rest, find its final velocity. **[2 marks]**

(d) Determine the maximum force during the collision. **[2 marks]**

(e) Calculate the average force during the contact period. **[2 marks]**

Section D: Three-Dimensional Momentum [25 marks]

Question 7 [25 marks] A bomb initially at rest explodes into four fragments. Fragment A (mass 2 kg) flies off with velocity $(15, 10, 5)$ m/s, fragment B (mass 3 kg) with velocity $(-8, 12, -6)$ m/s, fragment C (mass 1.5 kg) with velocity $(10, -15, 20)$ m/s, and fragment D has mass 2.5 kg.

(a) Find the velocity vector of fragment D using conservation of momentum. **[6 marks]**

(b) Calculate the magnitude and direction of fragment D's velocity. **[4 marks]**

(c) Find the total kinetic energy of all fragments after explosion. **[5 marks]**

(d) If the explosion lasted 0.0005 seconds, calculate the average force experienced by fragment A. **[4 marks]**

(e) Determine the impulse experienced by fragment C during the explosion. **[3 marks]**

(f) Verify that the center of mass remains stationary after explosion. **[3 marks]**

Section E: Advanced Rocket Dynamics [30 marks]

Question 8 [18 marks] A two-stage rocket has initial total mass 8000 kg. The first stage (mass 3000 kg including fuel) burns fuel at 45 kg/s with exhaust velocity 2200 m/s for 50 seconds, then separates. The second stage (mass 2000 kg including fuel) then burns fuel at 25 kg/s with exhaust velocity 3200 m/s for 60 seconds.

(a) Calculate the thrust force for each stage. **[4 marks]**

(b) Find the velocity gained during the first stage burn (ignore gravity). **[5 marks]**

(c) Calculate the mass remaining after first stage separation. **[2 marks]**

(d) Find the additional velocity gained during the second stage burn. **[4 marks]**

(e) Calculate the final velocity of the payload. **[3 marks]**

Question 9 [12 marks] A space probe performs a gravity assist maneuver around Jupiter. The probe approaches with velocity 20 km/s relative to the Sun and leaves with velocity 35 km/s relative to the Sun, both in the orbital plane. Jupiter's orbital velocity is 13 km/s.

(a) Calculate the probe's approach and departure velocities relative to Jupiter. **[4 marks]**

(b) Show that the collision with Jupiter is elastic in Jupiter's reference frame. **[4 marks]**

(c) Calculate the change in the probe's kinetic energy relative to the Sun. **[4 marks]**

Section F: Impulse in Continuous Systems [20 marks]

Question 10 [12 marks] Water from a fire hose strikes a wall perpendicularly at 25 m/s. The hose delivers 150 kg of water per minute. Assume the water comes to rest against the wall.

(a) Calculate the momentum change per unit time. **[3 marks]**

(b) Find the force exerted by the water on the wall. **[3 marks]**

(c) If the wall can withstand a maximum force of 5000 N, determine the maximum safe flow rate. **[3 marks]**

(d) Calculate the impulse delivered to the wall in 30 seconds of operation. **[3 marks]**

Question 11 [8 marks] A conveyor belt moving at 3 m/s receives sand dropped vertically onto it at a rate of 50 kg/s. The sand acquires the belt's horizontal velocity.

(a) Calculate the horizontal force required to maintain the belt's constant speed. **[4 marks]**

(b) Find the power required to overcome this force. **[2 marks]**

(c) Calculate the rate at which kinetic energy is gained by the sand. **[2 marks]**

Section G: Comprehensive Applications [20 marks]

Question 12 [20 marks] A sophisticated collision experiment involves three pucks on an air hockey table. Puck A (mass 0.3 kg) moves at 8 m/s due east and simultaneously collides with puck B (mass 0.4 kg) moving at 6 m/s due north and puck C (mass 0.5 kg) moving at 4 m/s due south. After the complex collision, puck A moves at 3 m/s at 30° north of east, and puck B moves at 5 m/s due east.

(a) Calculate the total momentum before collision. **[4 marks]**

(b) Find the velocity of puck C after collision using conservation of momentum. **[6 marks]**

(c) Calculate the kinetic energy before and after collision. **[5 marks]**

(d) Determine whether this collision scenario is physically possible. **[3 marks]**

(e) If the collision lasts 0.008 seconds, estimate the average force between the pucks. **[2 marks]**

Physics Data and Formulae

Momentum and Impulse:

Momentum: $\vec{p} = m\vec{v}$

Impulse: $\vec{J} = \vec{F}\Delta t = \Delta\vec{p}$

For variable force: $J = \int_0^t F(t')\,dt'$

Conservation: $\sum \vec{p}_{\text{before}} = \sum \vec{p}_{\text{after}}$

Center of Mass:

Position: $\vec{r}_{cm} = \frac{\sum m_i \vec{r}_i}{\sum m_i}$

Velocity: $\vec{v}_{cm} = \frac{\sum m_i \vec{v}_i}{\sum m_i}$

Force equation: $\vec{F}_{ext} = M\vec{a}_{cm}$

Collision Analysis:

Coefficient of restitution: $e = \frac{\text{separation speed}}{\text{approach speed}}$

Elastic (1D): $v_1' = \frac{(m_1 - m_2)v_1 + 2m_2 v_2}{m_1 + m_2}$

$v_2' = \frac{(m_2 - m_1)v_2 + 2m_1 v_1}{m_1 + m_2}$

Inelastic: $v' = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$

Rocket Equations:

Thrust: $F = \frac{dm}{dt} v_{exhaust}$

Velocity: $v = v_0 + v_{exhaust} \ln\left(\frac{m_0}{m}\right)$

For continuous ejection: $m(t) = m_0 - \frac{dm}{dt} \cdot t$

Energy Relations:

Kinetic energy: $E_k = \frac{1}{2}mv^2$

Gravitational PE: $E_p = mgh$

For pendulum: $h = l(1 - \cos\theta)$

Vector Operations:

Magnitude: $|\vec{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2}$

Dot product: $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$

Unit vector: $\hat{v} = \frac{\vec{v}}{|\vec{v}|}$

Calculus Relations:

$\frac{d}{dt}[t^n] = nt^{n-1}$

$$\int t^n \, dt = \frac{t^{n+1}}{n+1} + C$$

Constants:
Acceleration due to gravity: $g = 9.8$ m/s^2

END OF TEST

Total marks: 200

Grade boundaries: A* 180, A 160, B 140, C 120, D 100, E 80

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