A Level Statistics
Practice Test 2: Measures of Location and Spread

Instructions:
Answer all questions. Show your working clearly.
Calculators may be used unless stated otherwise.
Draw diagrams where appropriate to illustrate your solutions.
Time allowed: 3 hours

Section A: Advanced Central Tendency and Weighted Averages [25 marks]

1. **[12 marks]** Define and calculate advanced measures of central tendency:

(a) Define the geometric mean and explain when it's more appropriate than the arithmetic mean.

(b) Calculate the geometric mean of: 4, 9, 16, 25, 36

(c) Define the harmonic mean and provide a practical application.

(d) Calculate the harmonic mean of: 12, 18, 24, 36

(e) Explain the relationship: Harmonic Mean Geometric Mean Arithmetic Mean

(f) A car travels 60 km at 40 km/h and 60 km at 60 km/h. Calculate the average speed using the harmonic mean.

2. **[8 marks]** Calculate weighted averages and their applications:

(a) A student's final grade is calculated as: Coursework (30

(b) A company has three departments: Sales (40 employees, average salary £32,000), Technical (25 employees, average salary £48,000), Management (10 employees, average salary £65,000). Calculate the overall average salary.

(c) Explain why weighted averages are necessary in these contexts.

(d) If the Technical department's average salary increases to £52,000, calculate the new overall average.

3. **[5 marks]** Analyze the effects of data transformations on measures of central tendency:

(a) If all data values are multiplied by 3, how does this affect the mean, median, and mode?

(b) If 10 is added to all data values, how are the measures of central tendency affected?

(c) Given: mean = 45, median = 42. If all values are transformed by y = 2x + 5, find the new mean and median.

Section B: Variance, Standard Deviation and Standardization [30 marks]

4. [**15 marks**] Calculate and interpret measures of spread for complex datasets:

(a) Calculate the mean and standard deviation for: 85, 92, 78, 96, 88, 90, 82, 94, 86, 91

(b) Use the computational formula: $s^2 = \frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1}$ to verify your answer.

(c) Calculate the coefficient of variation and interpret its meaning.

(d) If each value is converted from centimeters to inches (divide by 2.54), calculate the new standard deviation.

(e) A second dataset has the same mean but standard deviation of 12. Compare the variability of both datasets.

5. [**15 marks**] Standardization and z-scores:

(a) Define z-scores and explain their purpose in data analysis.

(b) For the dataset in question 4(a), calculate z-scores for all values.

(c) Identify any values that are more than 2 standard deviations from the mean.

(d) A student scores 78 in Math (class mean = 72, SD = 8) and 85 in Science (class mean = 80, SD = 6). Which performance is relatively better?

(e) Calculate the standardized scores and interpret the results.

(f) Explain how z-scores allow comparison between different scales and distributions.

Section C: Percentiles and Advanced Box Plot Analysis [35 marks]

6. [**18 marks**] The following data represents reaction times (in milliseconds) for 30 participants:
245, 250, 255, 260, 265, 268, 270, 272, 275, 278, 280, 282, 285, 288, 290, 292, 295, 298, 300, 302, 305, 308, 310, 315, 320, 325, 330, 340, 350, 380

(a) Calculate the 10th, 25th, 50th, 75th, and 90th percentiles.

(b) Determine the interquartile range and semi-interquartile range.

(c) Use the 1.5 $\times$ IQR rule to identify outliers.

(d) Calculate the mean and standard deviation.

(e) Construct a five-number summary and create a box plot.

(f) Compare the mean and median - what does this suggest about the distribution shape?

(g) Calculate the percentage of data within one standard deviation of the mean.

(h) If the normal reaction time range is 250-300ms, what percentage of participants fall within this range?

(i) Comment on the distribution and identify any unusual features.

7. [**17 marks**] Compare performance between two groups using advanced statistical measures:
Group A (Morning): Q1 = 72, Median = 78, Q3 = 85, Mean = 79.2, SD = 8.5, n = 25 **Group B (Afternoon):** Q1 = 68, Median = 74, Q3 = 82, Mean = 75.8, SD = 9.2, n = 30

(a) Compare the central tendency between the two groups.

(b) Calculate and compare the coefficient of variation for both groups.

(c) Determine which group has more consistent performance.

(d) Estimate the range of the middle 50% of scores for each group.

(e) If a score of 85 was achieved in each group, calculate the z-score for each.

(f) Which group shows better overall performance? Justify your answer.

(g) Construct comparative box plots for both groups.

(h) Calculate the combined mean and standard deviation for all 55 participants.

Answer Space

Use this space for your working and answers.

Formulae and Key Concepts

Alternative Means:

Geometric Mean: $\bar{x}_g = \sqrt[n]{x_1 \times x_2 \times \ldots \times x_n}$

Harmonic Mean: $\bar{x}_h = \frac{n}{\frac{1}{x_1}+\frac{1}{x_2}+\ldots+\frac{1}{x_n}}$

Weighted Mean: $\bar{x}_w = \frac{\sum w_i x_i}{\sum w_i}$

Variance and Standard Deviation:

Sample variance: $s^2 = \frac{\sum(x-\bar{x})^2}{n-1}$ or $s^2 = \frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1}$

Population variance: $\sigma^2 = \frac{\sum(x-\mu)^2}{n}$

Standard deviation: $s = \sqrt{s^2}$ or $\sigma = \sqrt{\sigma^2}$

Standardization:

z-score: $z = \frac{x-\bar{x}}{s}$ or $z = \frac{x-\mu}{\sigma}$

Coefficient of Variation: $CV = \frac{s}{\bar{x}} \times 100\%$

Percentiles and Quartiles:

Percentile position: $P_k = \frac{k(n+1)}{100}$

Q1 position: $\frac{n+1}{4}$, Q3 position: $\frac{3(n+1)}{4}$

Semi-IQR: $\frac{Q3-Q1}{2}$

Linear Transformations:

If $y = ax + b$:

$$\bar{y} = a\bar{x} + b$$
$$s_y = |a| \times s_x$$
$$\sigma_y = |a| \times \sigma_x$$

Outlier Detection:

Mild outliers: $< Q1 - 1.5 \times IQR$ or $> Q3 + 1.5 \times IQR$

Extreme outliers: $< Q1 - 3 \times IQR$ or $> Q3 + 3 \times IQR$

Combined Statistics:

Combined mean: $\bar{x}_{combined} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$

Combined variance: More complex - requires individual sums of squares

Distribution Shape Indicators:

Mean ¿ Median: Positive skew (right-tailed)

Mean ¡ Median: Negative skew (left-tailed)

Mean Median: Approximately symmetric

Box Plot Components:

Five-number summary: Min, Q1, Median, Q3, Max

Box: Q1 to Q3, line at median

Whiskers: Extend to furthest non-outlier

Outliers: Plotted as individual points

END OF TEST

Total marks: 90

For more resources and practice materials, visit:
stepupmaths.co.uk